



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

		37	78	29	70	21	62	13	54	5		
	46	6	38	79	30	71	22	63	14	46	6	
		47	7	39	80	31	72	23	55	15	47	
		16	48	8	40	81	32	64	24	56	16	
		57	17	49	9	41	73	33	65	25	57	
		26	58	18	50	1	42	74	34	66	26	
		67	27	59	10	51	2	43	75	35	67	
		36	68	19	60	11	52	3	44	76	36	
		77	28	69	20	61	12	53	4	45	77	
		37	78	29	70	21	62	13	54	5	46	

AVERAGE AND PROBABILITY.

79. Proposed by the late ENOCH BEERY SEITZ.

Two equal spheres touch each other externally. If a point be taken at random within each sphere, show that (1) the chance that the distance between the points is less than the diameter of either sphere is $13/35$, and (2) the average distance between them is $11/5r$. [This is problem 5835, *Educational Times*, of London.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

(1) Let A , B be the centers and C the point of contact of the two spheres, each radius r .

From any point P in DC with a radius $=2r$ describe a sphere cutting B in Q , R . From B as a center with a radius BP describe a sphere cutting A in K , M . If P is the first point, the second point must fall within the double-convex lens $CQRC$. P may fall anywhere on the zone KPM and the second point must fall in a section of B equal to the double-convex lens $CQRC$.

From P as center with a radius $PS < 2r$ but $> PC$, draw the zone SLT . Let $DP = x$, $PS = y$, area of zone $KPM = 2\pi \cdot BP \cdot PG$, area of zone $SLT = 2\pi \cdot PS \cdot HL$.

$BP = 3r - x$, $AG = r - x - PG$, $BG = 3r - x - PG$, $PS = y$, $BH = 3r - x - y + HL$, $PH = y - HL$.

$$KG^2 = r^2 - (r - x - PG)^2 = (3r - x)^2 - (3r - x - PG)^2.$$

$$\therefore PG = x(2r - x)/4r.$$

$$SH^2 = r^2 - (3r - x - y + HL)^2 = y^2 - (y - HL)^2.$$

$$\therefore HL = [r^2 - (3r - x - y)^2]/2(3r - x).$$

$$\therefore \text{Area of zone } KPM = (\pi x/2r)(3r - x)(2r - x).$$

$$\text{Area of zone } SLT = [\pi y/(3r - x)][r^2 - (3r - x - y)^2].$$

Let p = chance, Δ = average distance.

$$\therefore p = \{ \pi^2 / [2r(\frac{4}{3}\pi r^3)^2] \} \int_0^{2r} x(3r-x)(2r-x) dx \int_{2r-x}^{2r} [y/(3r-x)][r^2 - (3r-x-y)^2] dy$$

$$= (3/128r^7) \int_0^{2r} (14rx^5 - x^6 - 48r^2x^4 + 48r^3x^3) dx = (3/128r^7)(1664r^7/105) = \frac{1}{3}\frac{3}{5}.$$

$$2. \Delta = \{ \pi^2 / [2r(\frac{4}{3}\pi r^3)^2] \} \int_0^{2r} x(3r-x)(2r-x) dx \int_{2r-x}^{4r-x} [y^2/(3r-x)][r^2 - (3r-x-y)^2] dy$$

$$= (3/40r^4) \int_0^{2r} (92r^3x - 106r^2x^2 + 40rx^3 - 5x^4) dx = (3/40r^4)(88r^5/3) = 11r/5.$$

80. Proposed by G. B. M. ZEER, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

A box contains 100 balls marked from 1 to 100. 13 balls are drawn at random. What is the chance that the balls marked from 1 to 10 are included in the 13 drawn?

Solution by J. W. YOUNG, Columbus, Ohio.

Since in all the favorable chances only three balls may vary, the total number of favorable chances is 9C_3 , i. e., the number of combinations of 90 things taken 3 at a time.

The total number of ways in which the balls may be drawn is, of course, ${}^{100}C_{13}$.

Hence the desired probability is equal to

$$\frac{{}^9C_3}{{}^{100}C_{13}} = \frac{\frac{90.89.88}{1.2.3}}{\frac{100.99.98.97.96 \dots 89.88}{1.2.3.5 \dots 13}} = \frac{1}{67515927540}.$$

PROBLEMS FOR SOLUTION.

ARITHMETIC.

124. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Science, Decorah Institute, Decorah, Iowa.

At what time between 5 and 6 o'clock is the minute hand midway between 12 and the hour hand? When is the hour hand midway between 4 and the minute hand?